Solution

1. (a) \((x - 3)(2x - 1) = 0\)
   \[x - 3 = 0 \text{ or } 2x + 1 = 0\]
   \[\therefore x = \frac{3}{2} \text{ or } x = -\frac{1}{2}\]

   (b) \(-4(2x + 3)^2 = 0\)
   \[(2x + 3)^2 = 0\]
   \[2x + 3 = 0 \text{ or } 2x + 3 = 0\]
   \[x = -\frac{3}{2} \text{ or } x = -\frac{3}{2}\]
   \[\therefore x = -\frac{3}{2} \text{ (repeated)}\]

   (c) \((3 - 4x)(-4 - 7x) = 0\)
   \[3 - 4x = 0 \text{ or } -4 - 7x = 0\]
   \[\therefore x = \frac{3}{4} \text{ or } x = -\frac{4}{7}\]

2. (a) \(x^2 + x - 12 = 0\)
   \((x + 4)(x - 3) = 0\)
   \[x + 4 = 0 \text{ or } x - 3 = 0\]
   \[\therefore x = -4 \text{ or } x = 3\]

   (b) \(6x^2 + 41x - 7 = 0\)
   \((6x - 1)(x + 7) = 0\)
   \[6x - 1 = 0 \text{ or } x + 7 = 0\]
   \[\therefore x = \frac{1}{6} \text{ or } x = -7\]

   (c) \(16x^2 - 4 = 0\)
   \[4(4x^2 - 1) = 0\]
   \[4(2x + 1)(2x - 1) = 0\]
   \[2x + 1 = 0 \text{ or } 2x - 1 = 0\]
   \[\therefore x = -\frac{1}{2} \text{ or } x = \frac{1}{2}\]

   (d) \(49x^2 + 42x = 0\)
   \(7x(7x + 6) = 0\)
   \[x = 0 \text{ or } 7x + 6 = 0\]
   \[\therefore x = 0 \text{ or } x = -\frac{6}{7}\]
3. (a) \[
\left( \frac{4}{3} - x \right)^2 = 16
\]
\[
\left( x - \frac{4}{3} \right)^2 = 16
\]
\[
x - \frac{4}{3} = \pm 4
\]
\[
x = \frac{4}{3} \pm 4
\]
\[
\therefore \quad x = \frac{4}{3} + 4 \quad \text{or} \quad \frac{4}{3} - 4
\]
\[
= \frac{16}{3} \quad \text{or} \quad -\frac{8}{3}
\]
(b) \[
(x + 3)^2 = 8
\]
\[
x + 3 = \pm \sqrt{8}
\]
\[
x = -3 \pm \sqrt{8}
\]
\[
\therefore \quad x = -3 + \sqrt{8} \quad \text{or} \quad x = -3 - \sqrt{8}
\]
4. (a) \[
3x^2 - 7x - 2 = 0
\]
\[
x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-2)}}{2(3)}
\]
\[
= \frac{7 \pm \sqrt{73}}{6}
\]
\[
\therefore \quad x = \frac{7 + \sqrt{73}}{6} \quad \text{or} \quad \frac{7 - \sqrt{73}}{6}
\]
i.e. \quad x = 2.59, \text{ cor. to 2 d.p.} \quad \text{or} \quad -0.26, \text{ cor. to 2 d.p.}
(b) \quad -4x^2 - 5x - 1 = 0
\[
x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(-4)(-1)}}{2(-4)}
\]
\[
= \frac{5 \pm \sqrt{25 - 16}}{-8}
\]
\[
= \frac{5 \pm 3}{-8}
\]
\[
\therefore \quad x = \frac{5 + 3}{-8} \quad \text{or} \quad \frac{5 - 3}{-8}
\]
\[
= -1 \quad \text{or} \quad -\frac{1}{4}
\]
5. (a) \((3x - 1)^2 - 4 = 0\)
   \((3x - 1)^2 = 4\)
   \(3x - 1 = \pm 2\)
   \(x = \frac{1 \pm 2}{3}\)
   \(\therefore \quad x = \frac{1}{3} \quad \text{or} \quad x = -\frac{1}{3}\)

(b) \(12x^2 - 12x - 9 = 0\)
   \(3(4x^2 - 4x - 3) = 0\)
   \(3(2x - 3)(2x + 1) = 0\)
   \(2x - 3 = 0 \quad \text{or} \quad 2x + 1 = 0\)
   \(\therefore \quad x = \frac{3}{2} \quad \text{or} \quad x = -\frac{1}{2}\)

(c) \(x = 0.686 \quad \text{or} \quad x = -2.19\)

6. (a) \(3 - \frac{1}{2}x + 3x^2 = -4\)
   \(3x^2 - \frac{1}{2}x + 7 = 0\)
   \(6x^2 - x + 14 = 0\)

(b) \(3 - \frac{1}{2}x + 3x^2 = 2x\)
   \(3x^2 - \frac{1}{2}x - 2x + 3 = 0\)
   \(3x^2 - \frac{5}{2}x + 3 = 0\)
   \(6x^2 - 5x + 6 = 0\)
7. 
(a) From the x-intercepts of the graph of \( y = -6 - x + x^2 \), the roots of the equation 
\(-6 - x + x^2 = 0\) are 3 and -2 approximately.

(b) Draw the line \( y = -4 \) on the graph. From the x-coordinates of the points of 
intersection between the graphs of \( y = -6 - x + x^2 \) and the horizontal line 
\( y = -4 \), we can see that the roots of the equation \(-6 - x + x^2 = -4\) are 2 and -1 
approximately.

(c) Draw the line \( y = 2 \) on the graph. From the x-coordinates of the points of 
intersection between the graphs of \( y = -6 - x + x^2 \) and the horizontal line 
\( y = 2 \), we can see that the roots of the equation \( x^2 - x - 6 = 2 \) are -2.4 and 3.4 
approximately.

8. (a) \[ -4x(2 - 3x) + 7 = 0 \]
\[ -8x + 12x^2 + 7 = 0 \]
\[ 12x^2 - 8x + 7 = 0 \]
\[ \Delta = (-8)^2 - 4(12)(7) = 64 - 336 = -272 \]
\[ \therefore \Delta < 0 \]
\[ \therefore \text{The equation has no real roots.} \]

(b) \[ 9(x + 2)^2 - 12(x + 2) + 4 = 0 \]
\[ 9(x^2 + 4x + 4) - 12x - 24 + 4 = 0 \]
\[ 9x^2 + 36x + 36 - 12x - 24 + 4 = 0 \]
\[ 9x^2 + 24x + 16 = 0 \]
\[ \Delta = 24^2 - 4(9)(16) = 576 - 576 = 0 \]
\[ \therefore \text{The equation has two equal real roots.} \]
9. \[2x^2 + 4x + a = 5\]
\[2x^2 + 4x + (a - 5) = 0\]
Since the equation has no real roots, the discriminant \(\Delta < 0\).
i.e. \[4^2 - 4(2)(a - 5) < 0\]
\[16 - 8a + 40 < 0\]
\[a > 7\]

10. (a) \[9x^2 - kx + 1 = x\]
\[9x^2 - kx - x + 1 = 0\]
\[9x^2 - (k + 1)x + 1 = 0\]
Since the equation has two equal real roots, the discriminant \(\Delta = 0\).
i.e. \[((-k + 1))^2 - 4(9)(1) = 0\]
\[(k + 1)^2 - 36 = 0\]
\[(k + 1)^2 = 36\]
\[k + 1 = \pm 6\]
\[k = \frac{5}{2} \text{ or } -7\]
(b) When \(k = -7\), the equation is \(9x^2 + 6x + 1 = 0\).
\[\therefore \ x = -\frac{6}{2(9)} = -\frac{1}{3} \text{ (repeated)}\]

11. Since the equation \((p + 4)x^2 + 2px + (p - 15) = 0\) has real roots, the discriminant \(\Delta \geq 0\).
i.e. \[(2p)^2 - 4(p + 4)(p - 15) \geq 0\]
\[4p^2 - 4(p + 4)(p - 15) \geq 0\]
\[p^2 - (p + 4)(p - 15) \geq 0\]
\[p^2 - (p^2 - 15p + 4p - 60) \geq 0\]
\[11p + 60 \geq 0\]
\[p \geq -\frac{60}{11}\]

12. Let \(x\) and \(x + 2\) be the two positive even integers.
\[x = \sqrt{x + 2}\]
\[x^2 = x + 2\]
\[x^2 - x - 2 = 0\]
\[(x - 2)(x + 1) = 0\]
\[x = 2 \text{ or } -1 \text{ (rejected)}\]
\[\therefore \text{ The two integers are 2 and 4.}\]

13. (a) \(AB = (x - 7) \text{ cm}\)
\[AC = (x + 1) \text{ cm}\]
(b) By Pythagora’s theorem, 

\[ AB^2 + BC^2 = AC^2 \]

\[ (x - 7)^2 + x^2 = (x + 1)^2 \]

\[ x^2 - 14x + 49 + x^2 = x^2 + 2x + 1 \]

\[ x^2 - 16x + 48 = 0 \]

\[ (x - 12)(x - 4) = 0 \]

\[ x = 12 \text{ or } 4 (rejected) \]

The perimeter of \( \triangle ABC \)

\[ = (12 + 5 + 13) \text{ cm} \]

\[ = 30 \text{ cm} \]

(c) \[ 2x^2 + 3x - 3 = 0 \]

\[ x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-3)}}{2(2)} \]

\[ = \frac{-3 \pm \sqrt{9 - (-24)}}{4} \]

\[ = \frac{-3 \pm \sqrt{33}}{4} \]

\[ \therefore \quad x = \frac{-3 + \sqrt{33}}{4} \quad \text{or} \quad \frac{-3 - \sqrt{33}}{4} \]

i.e. \[ x = 0.686, \text{ cor. to 3 sig. fig.} \quad \text{or} \quad -2.19, \text{ cor. to 3 sig. fig.} \]